Tutorial 4.

Preliminary:

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}, \quad a_{\overline{n}|i} = \frac{1-v^n}{i}$$

$$\ddot{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{d}, \quad \ddot{a}_{\overline{n}|i} = \frac{1-v^n}{d}$$

(Perpetuities: The infinite period annuity $(n \to \infty)$ is called a perpetuity.)

$$a_{\overline{\infty}|i} = \lim_{n\to\infty} a_{\overline{n}|i} = \lim_{n\to\infty} \frac{1-v^n}{i} = \frac{1}{i}. \quad \ddot{a}_{\overline{\infty}|i} = \lim_{n\to\infty} \ddot{a}_{\overline{n}|i} = \lim_{n\to\infty} \frac{1-v^n}{d} = \frac{1}{d}.$$

2.2.5.

j is 6-month interest rate, d_j is discount rate. $d_j = \frac{j}{1+j}$,

1. $\ddot{a}_{\overline{\infty}|j} = 20. \Rightarrow \frac{1}{d_j} = 20 \Rightarrow j = \frac{1}{19}.$

2-year rate $i = (1+j)^4 - 1 = (1+\frac{1}{19})^4 - 1$

$$X \ddot{a}_{\overline{\infty}|i} = 20 \Rightarrow X = 20 d_i = 20 \cdot \frac{i}{1+i} = 20 \cdot \frac{(1+\frac{1}{19})^4 - 1}{(1+\frac{1}{19})^4} = 3.71.$$

2.2.6.

Assume Sally's monthly payment is K, monthly rate is i

$10.000 = K a_{\overline{60}|i}$.

yield rate

1000. Tim Sally Bank

charge rate interest rate

saving account: $j^{(12)} = 6\%$, $j = 0.5\%$, month rate

yield rate: $k^{(2)} = 7.45\%$, $k = 3.725\%$ semiannual. rate.

Accumulate Value for saving account: $K s_{\overline{60}|0.5\%}$

$\quad\quad = 10.000(1+3.725\%)^{10} \Rightarrow K = 206.62.$

yield rate is define $L(1+k)^n = M$

L: initial investment, M: investment value

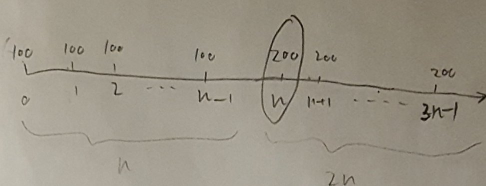
$10.000 = 206.62\, a_{\overline{60}|i} \Rightarrow i = 0.0073$ monthly rate. $\to$ nominal rate $i^{(12)} = 12i = 0.088.$

2.2.9.

seasonal rate $i = \frac{i^{(4)}}{4} = 0.04$. n is number of deposits.

$100\, \ddot{s}_{\overline{n|}0.04} = 200\, \ddot{a}_{\overline{2n|}0.04}$

$100 \cdot \frac{(1+0.04)^n - 1}{d_{0.04}} = 200 \cdot \frac{1 - v_{0.04}^{n}}{d_{0.04}}$



$v^{-n} - 1 = 2(1 - v^n)$

$v^n(v^{-n} - 1) = 2v^n(1 - v^n)$

$1 - v^n = 2v^n - 2v^{2n}$

$2v^{2n} - 3v^n + 1 = 0$

$(2v^n - 1)(v^n - 1) = 0$

$2v^n = 1$. $v = \frac{1}{1.04}$.

$v^n = \frac{1}{2}$.

$\left(\frac{1}{1.04}\right)^n = \frac{1}{2}$.

$(1.04)^n = 2$.

$n = \frac{\ln 2}{\ln 1.04} = 17.7$

18.